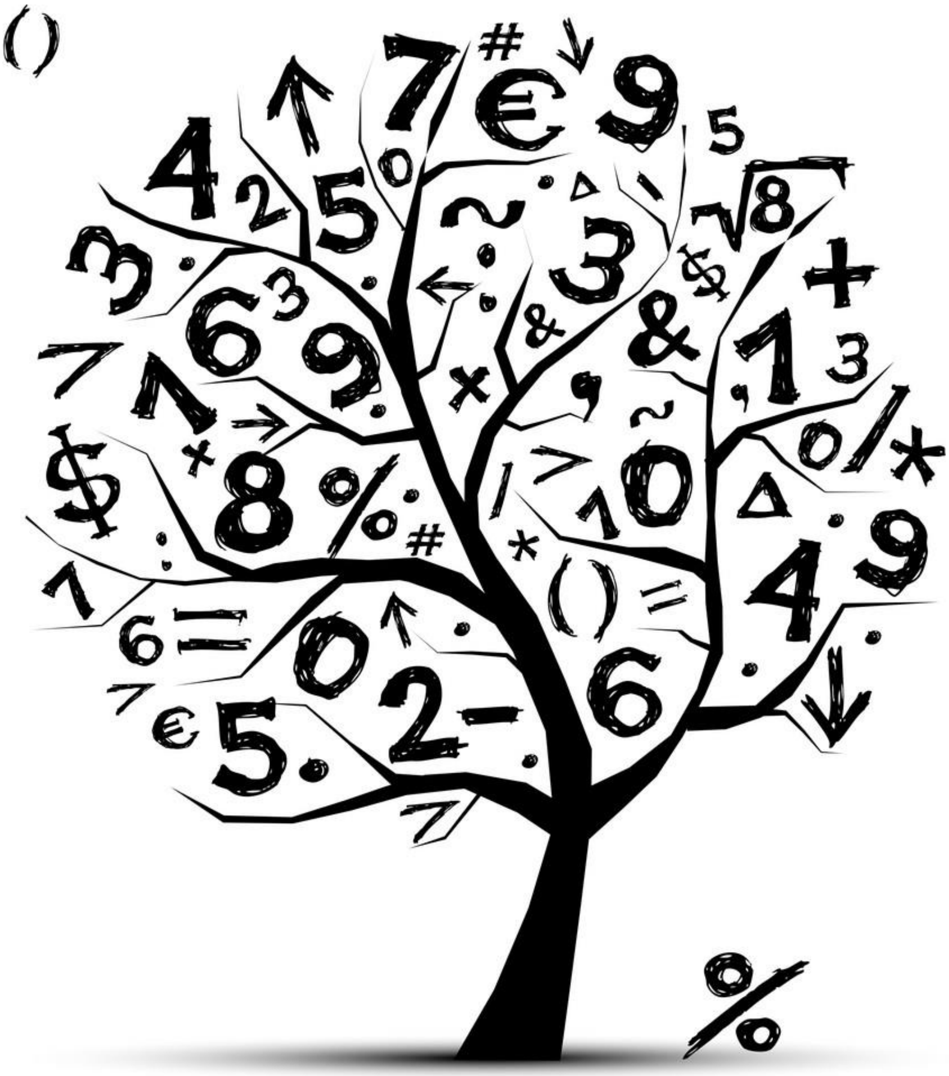
LONG ROAD SIXTH FORM COLLEGE CAMBRIDGE

A level Mathematics

Summer Work



Please submit this work on lined paper in your first maths lesson

Welcome to Mathematics A level. This work is designed to help you prepare for studying Maths by focussing on the hardest GCSE topics with a strong link to the A level syllabus. Before you start, please click on the link to view a video introduction to the summer work.

The content of this work will not be taught in lessons and we will assume you are able to do it well! So please do engage with it seriously to give you the best possible start on the course. We as teachers will mark this work and give you a grade for it – we will also use it to identify whether we can support you in any way to make the transition to the A level course go smoothly for you.

All the questions in this booklet are from past A level exams, and will cover:

- Indices
- Surds
- Linear Inequalities
- Quadratics (including completing the square)
- Modelling

If any of these topics are unfamiliar, you will need to do some independent work to help teach you these topics. You can find many questions to practise on the internet, but to help we've given you links at the start of each question to help give you an idea of the skills required. You might like to pause the videos to practise the basic skills yourself before tackling the harder exam questions in this booklet.

Indices

(Supporting videos here)

Show full working for each step of your working:

i) Express $125\sqrt{5}$ in the form 5^k [2]

ii) Simplify $(4a^3b^5)^2$ [2]

iii) Find the value of $\left(\frac{1}{25}\right)^{-\frac{1}{2}}$ [2]

iv) Simplify $\frac{(2x^2y^3z)^5}{4y^2z}$ [3]

v) Find the value of $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$ [3]

Surds

(Supporting videos here)

a) i) Express $\sqrt{45}$ in the form $n\sqrt{5}$, where n is an integer [1]

ii) Solve the equation

$$x\sqrt{20} = 7\sqrt{5} - \sqrt{45}$$

giving your answer in its simplest form [3]

b) Simplify fully the expression $\frac{6}{\sqrt{3}}$ [2]

c) Express $\frac{\sqrt{5}+3}{\sqrt{5}-2}$ in the form $p\sqrt{5} + q$, where p and q are integers [4]

Linear Inequalities

(Supporting videos here)

a) Solve the following inequality:

$$\frac{2x+1}{5} < \frac{3x+4}{6}$$ [4]

The width of a rectangular sports pitch is x metres, $x > 0$. The length of the pitch is 20m more than its width. Given that the perimeter of the pitch must be less than 300m,

b) form and solve a linear inequality in x [3]

Given that the area of the pitch must be greater than 4800m^2,

c) form a quadratic inequality in x [2]

Quadratics

(Supporting videos here)

Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0$$

a) By completing the square, express $f(x)$ in the form $(x-a)^2 + b$, where a and b are integers [3]

The curve C with equation $y = f(x)$, $x \geq 0$ meets the y-axis at P and has a minimum point at Q.

b) Sketch the graph of C, showing the coordinates of P and Q [4]

The line $y = 41$ meets C at the point R.

c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers [5]

Modelling

(This topic uses a mixture of previous skills)

A person throws a ball in a sports hall. The height of the ball, h m, can be modelled in relation to the horizontal distance from the point it was thrown from by the quadratic equation:

$$h = -\frac{3}{5}x^2 + 4x + \frac{7}{5}, \qquad x \geq 0$$

a) State the height that the ball was thrown from [1]

b) Calculate the horizontal distance travelled by the ball when it hits the floor of the sports hall [4]

The hall has a sloping ceiling with height, h m, which can be modelled by the equation:

$$h = 8 - \frac{1}{2}x$$

c) Determine, with reasons, whether the model predicts that the ball will hit the ceiling [4]

Problem Solving

This section of your summer work is intended to give you an opportunity to tackle some more open problems where there can be many different ways to get to an answer. This is a key skill that is needed in the new A level. Teachers will mark this work and give it back to you with a comment.

Problem 1

Alison has been exploring sums with surds. She used a spreadsheet to make columns for square roots, and then added together various combinations.

Here is one of the sums she worked out:

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{99}+\sqrt{100}}$$

The answer surprised her!

Can you find a way to evaluate the sum without using a calculator or a spreadsheet?

Can you find other similar sums with surds that give whole number answers?

Problem 2

We can define 2^{3^4} either as $(2^3)^4$ or as $2^{(3^4)}$. Does it make any difference?

Now calculate $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}}$ and $\sqrt{2}^{\left(\sqrt{2}^{\sqrt{2}}\right)}$. Which is the biggest?

Now consider the natural extension of both of these definitions to an infinite list of powers… $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}}$ Which definition would give the bigger value?